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TALMUDIC METROLOGY IX: BACK TO THE TALMUDIC MEASUREMENT UNITS OF VOLUME AND CAPACITY

A significant challenge in modern Jewish history research is the precise knowledge of the Talmudic measurement units of length, capacity, weight, and currency. Such knowledge is indispensable for the faithful to fulfil their religious, social, and economic obligations and to govern the Jewish society in the case of a lawsuit about agreed quantities. This problem was raised recurrently throughout history for two reasons. First, the same denomination is used in neighboring countries with different values, and second, the standard used in each country is unstable. It evolves frequently for political and military and economic reasons (debasement, change of the weight of currency). In a former paper (B.D.D. 19, January 2008) we showed that the Talmudic standards of length measurement units were presumably not based on natural values of breadth of thumb or of cubit but were based on the Roman mile equal to the Talmudic mile of 1481.5 m corresponding to 2000 military steps (or 2000*√2 cubits) and meeting the definition of the furthest distance one may walk on the Sabbath (*tehum shabbat*). In the present paper we examine the standard of Talmudical measurement units of volume and capacity. We find in the *Talmud Bavli* a rule defining the relation between the units of length and the units of capacity. Therefore, the Talmudic standard of length measurement units plus this relation allows defining the standard of capacity measurement units considered in the *Talmud Bavli*. However, we find in the *Talmud Yerushalmi* two different and contradictory rules which, added to the standard of length measurement units, allow defining two other standards of capacity measurements units. We are now faced

with the existence of three independent standards of capacity measurement units, which seem genuine and existed already around 200 C.E., at the time of the completion of the *Mishna* and the *Tossefta*. We can define them as respectively a standard of tall, average, and small units of volume. This would prove that the capacity denominations had a different value in different areas in Palestine. Apparently, the standard of the small units of volume of the *Talmud Yerushalmi* was still, in the eighth century, the only standard known by R. Eleazar Kalir, but later in the twelfth century, it was misunderstood by *Tossafot*, neglected, and practically completely forgotten. Similarly, the standard of the large units of volume was misunderstood, exaggerated, and probably therefore neglected and forgotten. Only the standard of the average units survived in Babylonia but the notion of the Roman mile and therefore the standard of the length measurement units was quickly forgotten in Babylonia. Finally, the only thing that remained was the relation between the Talmudic units of length and capacity. All the rabbinic standards of measurement units, generally based on the emulation of the natural sizes of the breadth of thumb or the hen's egg volume, respect this rule, known as the rule of Rav Hisda (see below note 10). We analyze four rabbinic standards of rabbinic measurement units, the standards of Rambam and Rashi and the standards defined recently in the former generation, by R. Naé and R. Avraham Isaiah Karelitz (*Hazon Ish*). The last one is a standard of tall units of length and volume. It corresponds to a late tendency expanding from the eighteenth century onwards and which experienced a lashing acceleration in the twentieth century, to privilege the natural value of the breadth of thumb. The two last standards are considered today as the authoritative halakhic standards. This is quite surprising because it appears that they were constructed on very imprecise and even incorrect assumptions and their authority can only be understood because the former standards were still insufficiently known. Their only merit is to put the faithful on the side of the security but at the price of significative exaggerations.

1. Preliminary Remarks

First: The present paper is a research paper aiming at the reconstitution of the Talmudic standard of units of measures. It has an historic and academic

interest but, in any manner, it hasn't any halakhic purpose. Whatever its results and conclusions may be, they will never go beyond the framework of an assumption. Even if its conclusion could be considered as a likely assumption, nearing certainty, it will probably not affect or touch the ritual rulings of *halakhah*. Indeed, it is a general accepted ruling enunciated by Maimonides that a court cannot overturn a decision taken by a former court, i.e., an enactment (*takana*: positive decision), a decree (*gezeira*: negative decision) or a conduct (*hanhaga*: *minhag*) with the only exception that it is greater than the former court in quality and quantity.[1] This rule concerns only enactments which were accepted and took root in all Israel. Otherwise, this enactment does not reach immutability and could be overturned by any court. Nevertheless, even local conducts (like special German or other local *minhagim*)[2] are much revered and difficultly overturn. Thus briefly, this paper aims at the reconstitution of the historic Talmudic reality, which must be studied without halakhic implication.[3]

Second: Today we note the existence of two halakhic standards of measurement units, which are taken into considerations in the application of

1 See Rambam, *Hikhot Mamrim*, chap. 2: 2, where he follows Mishna *Eduyot*, 1:5 where it says:

אין בית דין יכול לבטל דברי בית דין חברן עד שיהיה גדול ממנו בחכמה ובמניין.

This second court must be greater in wisdom and belong to a society including more scholars. Indeed, the court has a fixed number of members and cannot excel by the number of its members.

2 And each of the different halakhic standards of measurement units must be considered as a minhag which did not extend to all Israel. And indeed, there was never one standard, universally accepted.

3 Nevertheless, we note, from an historical point of view, that the halakhic metric system evolves: the standard of halakhic measurement units of *Hazon Ish,* more and more in common use today, is different and more stringent than the standard of halakhic measurement units in common use in the nineteenth century and even in the twentieth century before the war. These changes are difficult to explain, they are generally connected to the personality and the reputation of the Rabbi, the social influence of his students and the stringent atmosphere prevailing. Such an evolution was never induced by academic research. See below in note 101, the last visible stage of this slow and creeping evolution.

the ritual laws: the standard of the large measurement units of the *Hazon Ish*[4] and the standard of the small measurement units of Rabbi Avraham Hayyim Naé.[5] The existence of these two standards is the result of the forgetting of the Talmudic standard. They represent the most recent crystallization of two divergent traditions of understanding, which developed during the Middle Ages and the modern times, trying to emulate the Talmudic standard. There was indeed, at first glance, only one Talmudic standard,[6] but because of the forgetting, the rabbis failed finding a common understanding.

Third: The Talmudic sources teach us about the existence of different ancient historic standards of measurement units: the standard of measurement of Moshe Rabbeinu or *Midbari* (of the desert), the standard of Jerusalem[7] and the standard of Tiberias.[8] The Talmudic standard represents the final and centralized evolution. It tries emulating the standard of Moshe Rabbeinu, but it takes into account some rabbinic enactments. For example, the Talmudic Shekel weighs 20% more than the Shekel of the desert or the Shekel of Moshe.

4 Rabbi Avraham Yeshayahu Karelitz (1878-1953).

5 Rabbi Avraham Hayyim Naé (1890-1954).

6 This point will be discussed and adjusted later in this paper. I will prove that there were in fact, three different and independent standards of capacity measurement. Anyhow, in the field of surveying and distances, there was definitively only one standard of measurement units. Nevertheless, the different guilds of craftsmen used different cubits, apparently more or less different from the standard cubit. We find *ama shoheket, ama atsuvah, ama benonit, ama gedoumah, ama geroumah, ama metsumtsemet, ama atsilah*, two different standards in *Sha'ar Shushan* (the east door of the Temple mount). We find also, beside the *ama* of 6 handbreadths, the *ama* of 5 handbreadths for the utensils of the Temple. See *Erekh Milin,* Hayyim Jacob Sheftiel, Berditchov, 1907, for more details about these different denominations. This variety did not, apparently, disturb the unicity of the standard of measurement units in the field of surveying and measurement of distances.

7 See Ajdler (2009): *BDD* 21, "Talmudic Metrology III: Units of Measure of Volume and Capacity", p. 9. It concerned the units of capacity.

8 See Ajdler (2009) p. 10. It also concerned the units of capacity. Apparently still valid in the time of the Talmud. See Y. *Shabbat* 8:1 where it writes explicitly that this standard was still in use in the time of Rabbi Johanan.

2. Introduction

In a former paper Ajdler (2009),[9] we analyzed the problematic of the Talmudic units of measure of volume and capacity and especially the conundrum raised from the fifteenth century onward and later again with more acuity in the second half of the eighteenth century, resulting from the contradiction between the results of two contradictory methods of evaluation of the units of capacity, the evaluation of the volume of the capacities determined by the breadth of thumb and that determined by the volume of the hen's eggs.

Through lack of more objective data, the use of the natural values of the two following measures, the average breadth of thumb from one side and the use of the average volume of the hen's eggs were in use in the metrology of the Middle Ages and they lead to an internal contradiction. Indeed, the natural average breadth of thumb of adult men is estimated to 2.4 cm and the average volume of hen's eggs is about 50 cm^3. Now the fundamental formula binding the units of length and volume is given by the rule of Rav Hisda[10] according to which the volume of the Miqveh is 40 *seah* or 3 $(cubits)^3$. It is equivalent to the following formula:

> 1 *revi'it* = 1.5 egg = 10.8 E^3, where E is *etsba* and represents the breadth of thumb.

If we introduce E= 2.4 cm we get 1.5 egg = 1 *revi'it* = 149.29 cm^3 and 1egg = 99.53 cm^3. If we introduce 1egg = 50 cm^3 we get 1.5 egg = 75 = 10.8 E^3 and E = 1.9079 cm. In other words, the apparently natural values of E = 2.4 cm and 1 egg = 50 cm^3 are incompatible because they don't fulfill the rule of Rav Hisda.

From the end of the eighteenth century onwards, we had two main schools: those championing the small units of capacity, based on the volume of the natural hen's egg, and accepting the consequence of small units of length and those championing the large units of volume based on the current

9 Ajdler (2009): pp. 7–59. The present paper complements the former paper. An exhaustive bibliography can be found on pages 52–53 of that paper.

10 See B. *Pesahim* 109a, B. *Hagiga* 11a, B. *Yoma* 31a, B. *Eruvin* 4b and 14b.

average breadth of thumb, despite the consequence of a large volume of the eggs. Therefore, the latter school was obliged to accept the principle that originally, at the time of the Mishnah, the eggs were taller,[11] and their volume later diminished in the course of history.[12] A third small group[13] wanted, in order to solve the old conundrum mentioned above, to allow the coexistence between small units of capacity and bigger and normal units of length by considering a theoretical Mikveh of semi-ellipsoidal shape, with a circular basis of 1 cubit diameter and 3 cubits height. The first group followed the metrology of Maimonides or that of Rabbi Naé, while the second group followed the metrology of *Hazon Ish*.

In a former paper Ajdler (2008),[14] we adopted an original point of view: we developed a new theory about the Talmudic units of length used in surveying and measurement of distances, according, which the Talmudic units of length are not based on the average breadth of thumb of people, as it was certainly the case in the antiquity, but they had now an absolute definition. We proved based on different Talmudic quotations referring to controllable distances between known places,[15] expressed in miles that the

11 Nearly twofold.

12 There is in fact not the least evidence of such a strange evolution. On the contrary we have evidence that the size of the hen's eggs did not change significantly: 1. The weighing of Rabbi Hilaï Gaon. He found that the weight of an average hen's egg is 16.66 Dirham (of Babylonia) = 16.66 * 2.97 = 49.48 gr. 2. In the downfalls of Pompeii, ancient eggs were found. They had a volume of about 43 cm3. 3. Professor Zohar Amar from Bar-Ilan University succeeded in reconstituting an egg from pieces of the shell of an egg dating from 2600 years ago and found that its volume was about 42 cm3 and its weight was therefore about 45 gr. See more details about this last point at the following address:
https://www.ynet.co.il/articles/0,7340,L-5494311,00.html?fbclid=IwAR0rU1rRQ-MuJuhUiZDfNhjGEPZgARZ1Ro2hXbtMzsG7V0AMAqGOuUust2A

13 Mainly R. Jacob Gershom Weiss in his book: *Midot u mishkalot shel Torah,* Jerusalem, 1985.

14 Ajdler (2008): *BDD* 19, "Talmudical Metrology I: The Mile as a Unit of Length", pp. 55–83.

15 See Ajdler (2008). See also the private publications of Rabbi Yoel Shilo: *Shetilei Zeitim* and *Midat Mile.*
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Talmudic Mile is exactly the Roman Mile. The Roman mile was well known in Palestine; milestones could be found all through the country and some vestiges of them are still extant. The Mishnah Yoma 6: 4 seems to allude to the Roman mile (of 7.5 *ris*) being the *tehum* Shabbat[16] and a quotation in B. Eruvin 42a[17] refers to the evaluation of the *techum* Shabbat by 2000 steps, which seem to refer again to the Roman mile. The first reference binding the *tehum* Shabbat to 2000 cubits is found in Mishna Sota 5: 3. Rabbeinu Tam was the only Rabbi who correctly understood that we must understand: "the diagonal of 2000 cubits" so that the *tehum* Shabbat is 2828.4271 cubits in any direction.[18] Finally according to some authorities[19] the *tehum* Shabbat is 2828.43 cubits, hence the diagonal of a square of 2000 cubits side.

Therefore, it makes sense that the Roman Mile is the diagonal of a square of 2000 cubits side, which can be oriented in any direction.[20] Although the *halakhah* adopted the domain of Shabbat in the shape of a square of 2000 cubits side oriented along the cardinal directions, we find in *Hayé Adam* a reference to R. Tam's opinion.[21] According to the last estimation, the length

16 The walkable distance on Shabbat.

17 The statement of Rav Nahman in the name of Samuel: the *tehum Shabbat* is a walk of 2000 average steps. The adjective "average" seems to have troubled the rabbis who understood small steps, smaller than the normal. This led them to understand small steps equal to cubits. This is of course not a normal pace. It corresponds to the walk of an ill man who can hardly drag himself along. It should not be forgotten that at that time, all the population, except wealthy people, walked and covered long distances. Their average steps corresponded to about 74 cm (by contrast with the modern degenerated walker) and the distance of Sabbath defined by Rav Nahman corresponds to the Roman mile and sustains the theory of the Talmudic standard of length measurements units, based on the Roman mile and independent from the natural value of the breadth of thumb. The goose step of parading soldiers can reach more than 90 cm and is of course not an average step.

18 For the opinion of Rabbeinu Tam see B. *Eruvin* 51a, Tossafot כזה יהו כל שובתי שבת and B. *Yoma* 67a Tossafot וכולן על ידי עירוב.

19 See Hayé Adam, R. Abraham Dantzig, Vilnius 1810, *Hilkhot Shabbat* 76:2.

20 See B. Eruvin 51a and Ajdler (2008) pp. 65–66.

21 He notes that some gaonim consider that the *tehum* is 2828 cubits in all the directions and he expresses the opinion that in a case of emergency one could rest on this lenient opinion.

of the Roman Mile is 1481.5m,[22] and therefore the Talmudic cubit is 52.38 cm and *etsba*,[23] the breadth of the thumb is 2.1825 cm.[24]

These Talmudic units of length depend thus on the Roman Mile, which the Talmudic Rabbis adopted for their Mile.[25] As long as the Jewish people lived under Roman rule, this situation was evident. But later, when the main part of Judaism lived under Persian rule, this was forgotten, and they definitively confused the *tehum* Shabbat with 2000 cubits so that they finally equated the length of a step with a cubit![26] Our assumption about the Talmudic cubit is confirmed by archeological findings. The examination of the archeological remnants of the Temple and the Temple Mount and the comparison with the measurements given in the Mishna *Midot* represent another way to determine the Talmudic units of Length. Asher Grosberg has noted that the cubit which best fits the agreement between the archeological remains and the descriptions and measures from Mishna Midot and Josephus, is a cubit of about 52.5 cm.[27] Other archeological remains confirm this order of size of the Talmudic cubit. Daniel Levi summarized various archeological data confirming that the cubit ranged between 52 and 52.5 cm.[28]

In Ajdler (2009), we analyzed several Talmudic passages proving that all the Talmudic measurement units emulated the corresponding Roman

22 According to the accessible data the length of the Roman mile ranges between 1473m and 1482m with a preference in the 19th century for 1478m and today for 1481.5m: In this paper we adopted 1481.5m. But it should be remembered that there is a little degree of incertitude.

23 The cubit is equal to 6 *tefah* or handbreadth and to 6* 4= 24 *etsba* or thumb.

24 These values of *ama* and *etsba* are very close to those used by the rabbis of the nineteenth century. In Russia, the Rabbis equated 2000 cubits to the *vestre* of 1067m and therefore the *ama* was about 53 cm. See *Arukh ha Shulhan Yoreh Deah*, 201:3. See also *Kitsur Shulhan Arukh*, supplement *Shiurei ha-Mitsvot* by Rabbi David Feldman, entry: *tehum* Shabbat. See also Benish (1987): *Midot ve- Shiurei Torah* p. 91 for other references.

25 Probably after a slight adaptation of their ancient units of measure in order to fit the Roman standard.

26 Without reacting, despite the improbability of this relationship!

27 Asher Grosberg: *Ittur Tehumei har ha-Bayit u Makom ha-Mikdash*, Tehumin 26, 5756.

28 Daniel Moshe Levi: *Pitron hidat ha-shiourim al-pi ha-archeologia*, Tehumin 30, 5770, pp. 420–421.

units of measure, whether we deal with the units of length, of weight or of capacity. Different Talmudic quotations going back to the period of the Mishna, give evidence of this fact, which was probably a little adaptation of their ancient units of measures, slightly different, to fitting the Roman units of measure. This is certainly the case for the units of measure of weight as the following statement shows:

כיכר דבלה של שישים מנה באיטלקי, שביעית א׳, ב׳ ו א׳, ג׳.

This is also the case for the units of measure of capacity as the following statements show.

מדות הלח והיבש, שיעורן באיטלקי, זו מדברית, כלים י״ז: י״א.

חצי לוג יין באיטלקי, משנה סנהדרין ח׳: ב׳, ירושלמי סנהדרין מ״א ע״ב (דפוס וילנא).

וכולם במדה האיטלקית, תוספתא כתובות ה׳: ז׳.

רביעית יין באיטלקית, עירובין ס״ד ע״ב, ירושלמי עבודה זרה פ״א ה״ט, ויקרא רבה ל״ז.

The first quotation above-mentioned shows that they considered the Italian measures equivalent to the measures of the desert, thus the units used by Moshe Rabbeinu.[29] Clearly the standard of the Talmudic measurement units of volume and capacity was similar the Roman units of capacity. Similar but not identical because the proportion between two consecutive units in the sequence of the Talmudic and Roman units of capacity is not the same. This lets an area of incertitude in the attribution of the Roman correspondents of the Talmudic units of capacity. The comparison of both sequences of units of capacity maintains a doubt about the Roman attribution of the Log: is it the Sextarius of about 0.547 liters or the Hemina of about 0.274 liters? This hesitation and even confusion is confirmed and amplified in the extant translations of the Bible by the Septuaginta and Jerome and in the testimonies of Josephus. The choice of the first solution would give us the solution of the large Talmudic units of capacity and the choice of the

29 See the commentary of R. Samson ben Avraham of Sens on Mishnah *Shevi'it* I: 2.

second solution would give us the solution of the small Talmudic units of capacity. We based ourselves on different Talmudic passages which seem to accredit the theory that the Talmud followed the theory of the large units of capacity. We quote below two Talmudic passages supporting this thesis. The first following quotation supports the principle that Log corresponds to Kestes, the Greek denomination parallel to the Roman Sextarius. The second quotation seems to indicate that the Talmud identifies the *revi'it* with the Quartarius, the fourth part of the Sextarius.

קסתא דמורייסא דהווה בציפורי היא הוות כמין לוגא דמקדשא ובה משערין רביעית של פסח, פסחים ק״ט ע״א.

רביעית יין באיטלקית, עירובין ס״ד ע״ב, ירושלמי עבודה זרה פ״א ה״ט, ויקרא רבה ל״ז.

Therefore, we considered in the above-mentioned paper the principle that the Talmudic standard of measurement units was based on the units of length derived from the Roman Mile giving a Cubit of 52.38 cm and an *etsba* of 2.1825 cm from one side and units of capacity derived from the relation 1 Log = 1 Sextarius = 547 cm^3 from the other side. If we submit our new standard of unity of measure of volumes, based on an *etsba* of 2.1825 cm and a Log of 0.547 liters to the test of the fundamental formula[30] binding the units of length and volume:

$$1.5 \text{ egg} = 1 \textit{ revi'it} = 10.8 \text{ E}^3,$$

we find: 1.5 * 91.1667 = (547/4) = 136.75 cm^3 and 10.8 * $(2.1825)^3$ = 112.2757 cm^3.

Thus, the complete standard that we achieved, which should fulfill the Talmudic requirements and ensure a correct cubit and a correct capacity for the Log, does not satisfy the fundamental formula binding the units of length and volume. Furthermore, the volume of the average hen's egg corresponding to this standard of measurement is 91.1667 cm^3, which is nearly twice the volume of an average egg.

30 The formula of Rav Hisda, *Pesahim* 109a.

To solve the conundrum resulting from the contradiction between these large Talmudic units of capacity and length we proposed to consider that the volume of the eggs considered is not the volume of the egg in the strict sense, but the volume of the parallelepiped circumscribed to the egg. According to this conception, a volume of 144 eggs would not mean that it represents the volume of water displaced by this number of eggs, but it would mean a volume in which it is possible to store 144 eggs of an average size. Therefore, a volume of 91.1667 cm^3 for an egg corresponds to a real egg of 47.73 cm^3.[31]

What about the problem of the fundamental formula binding the Talmudic unities of length and capacity, once we are certain that these values E = 2.1825 cm and Log = about 547 cm^3 are the correct ones, we must ask ourselves, if this formula 1.5 eggs = 1 *revi'it* = 10.8 E^3 is not

31 See Ajdler (2009) pp. 41 – 42. Indeed: 91.1667 * (π / 6) = 47.7348 cm^3.
The volume of the ellipsoid (height: h, radius of circular base r) is: (2/3) pi*h*r^2
The volume of the circumscribed parallelepiped is 4*h* r^2
The density of the stacking is then pi/6. A scholarly reader objected that according to the Kepler conjecture it is possible to increase the density of the stacking to pi/3√2 by the "cannonball" packing, in the shape of a prism. Now, in fact, we are filling up a rectangular box with eggs, so that this objection is meaningless. We are not stacking the eggs in the shape of pyramids to increase the density of stacking! Furthermore, it doesn't seem possible to stack 144 balls by the cannonball packing. The wholesalers in eggs have always filled their boxes on the same manner: they stacked the same layers of eggs the one of the other and this was the best manner to maximize the number of eggs per box. In principle it is possible to increase the density of the stacking by seeing to it that each egg of the second layer lies between four eggs in the first layer and so on with other layer. But this is only true when the dimensions of the layers are very tall, theoretically infinite. When we stack eggs in a box with 6 * 6 eggs per layer, in 4 layers then the gain of stacking the second layer as mentioned above, is widely over-compensated by all the eggs that cannot be stacked along the periphery of the second and the fourth layer. Therefore, although the maximum density of a stacking can theoretically reach pi/3√2, practically, the volume in which it is possible to store 144 eggs, is 6/pi times their volumes.
A more serious objection to this assumption is the fact that the Mishna *Kelim* 17: 6 gives indications related to the measurement of the volume of an egg by weighing the volume of displaced water. Maimonides explains it more in detail in his commentary. We will answer this objection later.

- an approximation
- or if this is an ancient formula, which was not adapted when the Rabbis slightly adapted the ancient Rabbinical standard of measure to the Roman standard of measure.
- Or if perhaps the fundamental formula binding the Talmudic unities of length and capacities of Rav Hisda refers to another standard and does not fit our propounded standard of large units of capacity based on Log = about 0.547 liters and of units of length based on *etsba* = 2.1825 cm.

In the present paper we will adopt a completely new position. Instead of considering that there was one unique Talmudic system of capacity measurement, which we try emulating today by the standard of *Hazon Ish* or by that of Rabbi Naé, we will show that in fact there were apparently three different standards of Talmudic measurement units of volume and capacity, but only one undisputed principal standard of measure of length. These three standards of units of volume are described and mentioned in the Talmudic literature and they even find their roots in the Mishnah or in the Tossefta. Each of these three standards, had necessarily a different formula binding the units of length to the units of capacity. That means that these three standards gave to the same denomination of a capacity unit, another content. We will see that these three different standards already coexisted by about the year 200 CE, the time of the completion of the Mishnah and the Tossefta. The study of these three standards will allow us to better understand the subject and reconstruct the Talmudic reality.

Two of these standards would later completely disappear and be forgotten and only the third standard, the one which Rav Hisda described, would survive. This is at variance with the generally accepted opinion that there was only one standard of unity of measure of volume and capacity, the standard described by Rav Hisda in B. *Pesahim* 109a. In fact, the details of this third standard were also forgotten with time, as in fact were so many other biblical and rabbinical laws and traditions, except the binding formula of Rav Hisda. The forgetting of the elements of this ancient standard of measurement units of volume transformed the formula of Rav Hisda from the definition of one

special standard of units of capacity it originally had, into a general formula ruling the Talmudic measurement standards and binding the units of length to the units of capacity. Therefore, the champions of the standard of the large Talmudic units (standard of *Hazon Ish*) and those of the standard of the small Talmudic measures (Rabbi Naé) had to fulfill the fundamental formula of Rav Hisda. Both standards have their champions, and each standard has its variants and presents its difficulty and contradiction.

In fact, there is not a perfect standard because each of them has its weak sides and its strong sides. There is very abundant rabbinical literature on this subject ranging from the period of the Gaonim until the modern times. Its access and understanding are difficult because the reference units of length, weight and of volume used by the rabbis all through the history are difficult to specify. They are often bearing the same denomination, but their value depends on the country and epoch so that it is very misleading. The difficulty is further increased and decupled when it concerns coins, which are often used as units of weight and therefore also as units of volumes, calculated as the weight of water contained. The same denomination generally has a different weight at different periods and in different areas, and it evolves so fast because of the debasement of the coinage consequence of the financial difficulties of the states.

3. The Standard of the Talmudical Large Measurement Units of Volume and Capacity[32]

This standard was mentioned in Mishnah *Terumot* X: 8 and explained in Yerushalmi *Terumot* X: 5.[33] It concerns the quantity of unclean[34] fish that forbids a pickle of fish. The text of the Mishnah is as follows:

32 See Ajdler (2009) pp. 20–21. See also Table 1 towards the end of the paper.

33 According to the numeration of the *Talmud Yerushalmi* edited in Vilnius by Romm.

34 Non-kosher fish.

משנה:

דג טמא שכבשו עם דג טהור, כל גרב שהוא מחזיק סאתים, אם יש בו משקל עשרה זוז ביהודה שהן חמישה סלעין בגליל, דג טמא , צירו אסור...

גמרא:

כל גרב שמחזיק סאתים: כמה סאתה עבדא? עשרים וארבע לוגין וכמה לוגא עביד? תרתין ליטרין וכמה ליטרא עבדא? מאה זוזים, נמצא כל זין וזין אחת מתשע מאות ושישים...

It concerns the weight of non-kosher fish which makes unfit a pickle of two *se'ah* of kosher fish. As soon as the quantity of non-kosher fish reaches the weight of 10 Zuz, the whole pickle is unfit. The Zuz is a denomination equivalent to the dinar. The Dinar corresponds to the Roman Denarius. It is a unit of weight of 3.411 gr and the *se'ah* is a volume of 24 Log. The Zin is a denomination equivalent to 10 Zuz. The *gemara yerushalmit* tells us that each Log weighs 2 Litra and each Litra weighs 100 Zuz. Thus, the pickle has a volume of 48 Log and each Log of pure water weighs 200 Zuz. Finally neglecting the difference of density of pure water with regard of the pickle of fish, the weight of this pickle is 2 * 24 * 200 = 9600 Zuz.

Therefore, the proportion of non-kosher fish making the whole mixture unfit for consumption is 10/9600 or 1/960. Thus, according to the *Talmud Yerushalmi*, 1 Log of water weighs 200 Zuz. If we consider, according to the data of the Roman units of weight that 1 Zuz weighs 3.411 gr, then the weight of 1 Log is 3.411 * 200 = 682.2 gr and the Log has a capacity of 682.2 cm^3.

Now if the Zuz represents the coin of 1 Dinar, the fourth part of the Shekel according to the Tyrian standard, then its weight is slightly greater and weighs about 14.16 gr/4 =3.54 gr and the Log water weighs 3.54 * 200 = 708 gr and has a capacity of 708 cm^3. Both these values, 682.2 cm^3 or 708 cm^3, dependent on the weight adopted for the Zuz, are huge, much greater than any value proposed for the Log, Sextarius or Hemina, and even greater than the volume of 600 cm^3 adopted by *Hazon Ish*.

As already mentioned, the comparison of the standard of the Talmudic units of volume with that of the Roman units of volumes maintains a doubt

about the Roman attribution of the Log: Sextarius = 0.547 liters[35] or Hemina = 0.274 liters. The volume ascribed to the Log is thus exaggerated and the only plausible solution is that the *Talmud Yerushalmi* was dealing with the *se'ah* and the Log of Jerusalem.[36] Therefore, the Log *midbari*[37] weighs only (5/6) 200 = 166.667 dinars and, probably with more precision, **160 dinars**.[38] The weight of 1 Log *midbari* of water is then 3.411 gr * 160 = 545.76 gr which corresponds with a very good precision to the Sextarius of 0.547 l. We can conclude that the Mishna Terumot and the Talmud Yerushalmi work according to the standard of the large measurement units of volume. Let us now examine the formulas binding the units of length and capacity:

First formula:

1 Mikveh = 40 *se'ah* = 960 Log = 960 * 0.54576 = 523.93 l = k_1 * 143.713 l.[39]

We deduce: k_1 = 3.6457 instead of 3 in the classical formula:

35 Boeckl (1838) established the capacity of the sextarius to 545.75 but the most recent value adopted for the sextarius is 547 cm3.

36 See Ajdler (2009) pp. 9–10. The units of capacity of Jerusalem were (6/5) of the units of capacity of the desert

37 Of the desert or of Moses.

38 We should not be that surprised by this result. First, we have seen that there is a margin of error and uncertainty about the capacity of the Sextarius; second, there is a margin of error in the calculation made in *Yerushalmi Terumot*. Indeed, there is an approximation in this calculation because the density of the pickle was taken as 1. It is possible that the exact calculation of the *Yerushalmi* should have been the following. The equation: 1 Litra = 100 Zuz is an approximation. In the Roman metrology 1 Libra = 1 Talmudical Litra = 96 Denarius and 1 Mina = 100 Denarius and by approximation and confusion we find 1 Litra = 100 Denarius. If so the Log of Jerusalem of pure water would weigh 200 * 0.96 = 192 Zuz but 1 Log of the pickle with a density of about 1.0417 would finally weigh 200 Zuz. If so the weight of 1 Log *midbari* of water would be (5/6) * 192 = 160 Zuz weighing 160 * 3.411 = 545.76 gr if 1 Zuz = 3.411 gr (theoretical weight according to Roman metrology) or 160 * 3.50 gr = 560 gr if 1 Zuz = 3.50 gr and 160*3.54 = 566 gr if 1 Zuz = 3.54 gr (weight of the coin Dinar), according to the Tyrian standard. As we see there will always remain a possible margin of error of about 4%. We adopted the smallest value for the only reason that it gives a Log close to the Sextarius.

39 According to the standard of Talmudic units of length that we adopted: The Cubit (Amah) is C = 52.38 cm and C^3 = 143.713 dm^3.

1 mikveh = 40 *se'ah* = 3 (Cubit)^3.[40]

The formula is now: **1 mikveh = 40 Se'ah = 3.6457 (cubit)^3. (1)**

Second formula:

1 *revi'it* = k_2 (*etsba*)^3. Thus 136.44 = k_2 (2.1825)^3 = k_2 10.3959.

We deduce: k_2 = 13.1244 instead of 10.8 in the classical formula:

1 *revi'it* = 10.8 (*etsba*)^3.[41]

The formula is now: **1 *revi'it* = 13.1244 *(etsba)*^{3}. (2)**

Note that formulas (1) and (2) are equivalent. If we multiply formula (2) by 3840 we get formula (1).[42]

Let us now consider the theoretical minimal Mikveh of one cubit by one cubit section. It contains 523.93 l, and the height of the water is 3.6457C = 3.6457 * 0.5238 = 1.90 m. If an average man of 75 kg submerges, the apparent volume of water is 523.93 + 75 = 598.93 l, and the apparent height of the water is 598.93 l / (5.238)^2 dm^2= 21.82 dm = 2.18m. Thus, any man can submerge vertically without any problem.[43]

Strong points of this standard of the large Talmudic units of volume.

- Different passages in the Talmud, beginning with Mishna *Kelim* 17:11, show that the standard of the Talmudic units of measure of volumes and capacity was bound to the Roman standard. Nevertheless, both standards are not parallel so that a doubt remains whether the Log must be equalized to the Sextarius (or Kestes in Greek denomination) or Hemina[44] (Kyathos in Greek denomination).
- There are some Talmudic passages, which support that the Log was in fact equal to the Greek denomination Kestes and corresponded to the

40 *Eruvin* 4b and 14b, *Pesahim* 109b, *Yoma* 31a and *Hagiga* 11a.

41 The formula of Rav Hisda, *Pesahim* 109a.

42 (1/3840) *3.6457 (Cubit)^3 = (1/3840) *3.6457 * 13824 (*etsba*)^3 = 13.1244 (*etsba*)^3

43 We suppose a container of one cubit by one cubit section and sufficiently high to hold the water when a man submerges.

44 Hemina is the half of Sextarius.

larger denomination and that the *revi'it* corresponded to the Quartarius, the fourth part of the Sextarius.

Weak points of the standard of the large Talmudic units of volume.

- According to the former conclusions 1 *revi'it* is 136.44 cm^3 and *beitsah*, the egg is then 90.96 cm^3. This is not the volume of an average hen's egg. On the contrary, this corresponds to an exceptional extra-large egg.
- This is a serious problem. The solution, which was proposed, consists in measuring the volume of a parallelepipedal box in which it is possible to store the considered number of eggs.[45]
- Nevertheless, the Talmud mentions, even if briefly and nearly in an allusive way, the measure of the volume of eggs by measuring the volume of displaced water.[46] But this is not a refutation of our proposition. According to the champion of the large units of capacity, a volume expressed in eggs represents the number of eggs stored in this volume. This seems the only likely explanation. Now the volume of the parallelepiped circumscribed to the egg, which represents its litter is 6 / π times the volume of the egg or about twice the volume of the egg. Therefore, the measure of the volumes expressed in eggs divided by 2 gives the number of eggs stored in the volume. Thus, whatever the definition of the measure of a volume in eggs may be, we must always use the traditional method of measure of the volume of the eggs.
- The standard of the large units of volumes raises difficulties in the explanation of the *Yam shel Shelomo* and its contents of 2000 Bat. (1 Kings 1: 23).

Finally, we must note that this standard of large units of capacity is undeniably and completely described in the Mishna and *Talmud Yerushalmi* and constitute a coherent standard of units of capacity. It was nevertheless not correctly understood and was generally ignored.[47]

45 See Ajdler (2009) pp. 41–42 based on B. *Eruvin* 83b. This solution was proposed by Y. Borenstein.

46 See Mishna *Kelim* 17: 6 and commentary of Maimonides and Tossefta *Nazir*, 4: 1.

47 Rabbi Yom Tov Lipman Heller noted in *Ma'adanei Yom Tov*, Berakhot III:n° 30 and 80,

Maimonides in his commentary on this Mishna, copied, without any reservation, the words of the *Talmud Yerushalmi*, namely that the Log is two Litra and the Litra weighs 100 Zuz. Nevertheless, the opinion of Rambam is that Litra weighs 35 Zuz = 35 * 4.25[48] =148.75 gr. and has a capacity of 148.75 ml. Similarly, he gives for Log a weight of 70 Zuz = 70 * 4.25 gr. = 297.5 gr. and has a capacity of 297.5 ml. Maimonides' silence is striking.[49]

that the volumes of Y. *Terumot* X: 5 are three times the small volumes of Maimonides or more precisely 100/35 = 2.8571. In this fraction the numerator is the approximate weight of Litra see above) and the denominator is the weight of Litra according to Maimonides. In fact, because of many approximations, exaggerations and imprecision, the theoretical ratio between the large and the small units of measure of capacity is in fact given by the fraction 13.1244 /7.3333 = 1.7898, where the numerator is the coefficient of the *revi'it* of the standard of the large measures, formula (2) (see above). The denominator is the coefficient of the *revi'it* of the standard of the small measures, formula (3) (see below). The theoretical ratio is thus 1.8. The overvaluation results from two independent factors: 1. The numerator 100 approximates 96 and corresponds to the units of Jerusalem. In the units of the desert, it should be 96 * (5/6) = 80 (see above note 38). This already reduces the fraction to 2.28. 2. The denominator of 35 corresponds to one Litra of 35 Zuz, used by Maimonides, it is exceptionally small. As the ratio (tall units/small units) ~ 1.8, the litra adopted in Maimonides' standard of small units should be about 80/1.8 ~ 44 Zuz instead of 35. In fact, the figure 35 is the result of the weighing of Rabbi Hilaï Gaon (1 Litra = 2 revi'it = 3 Beitsah = 3 *16.66 = 50 Babylonian Dirhan = 50 * 0.7 = 35 Dinar) and of the choice of the Dinar of 4.25 gr. If Maimonides had known that the Dinar is only about 3.4 gr, then the weighing of R. Hilaï Gaon would have given 1.25 * 17.5 = 21.88 Dinar and Maimonides' Litra would have been 43.75 Dinar (with a Dinar of 3.54 gr the weight of the Litra would be 1.2 * 17.5= 42 Dinar). We have now the detailed explanation of the origin of the exaggerated ratio of 3 between the standards of the tall units of the Mishna and the standard of the small units of Maimonides. These facts seem to have escaped to R. Yom Tov Heller. Because of the impression of exaggeration, this passage in Y. *Terumot* wasn't generally taken seriously. It was considered as an individual and exaggerated position and was not followed by the rabbis and rulers.

48 The weight of the Dinar according to Maimonides and the Gaonim.

49 Although Maimonides corrected and changed his commentary on this Mishna *Terumot* X:5 (see Mishna with Maimonides' commentary, Mossad ha-Rav Kook 1963), on this point, he did not change. It is difficult to understand that Maimonides did not react about this overvaluation of 100/35=2.8571. Also note the silence of Maimonides' commentators on *Hilkhot ma'akhalot assurot* 15:34.

4. The Standard of the Talmudical Small Measurement Units of Volume and Capacity[50]

According to the principle that the standard of the Talmudic units of volume and capacity corresponds to the Roman standard, there is a second possibility to examine, that the Log is equal to Hemina and represents a volume of about 274 cm^3 and therefore we should find something like:

40 *se'ah* = 960 Log = 960 * 0. 274 = 263.04 l. Therefore:

1 Mikveh = 960 Log = 263.04 dm^3 = 1.83 * (143.713) = 1.83 $(cubit)^3$.

1 *revi'it* = 274/4 = 68.50 dm^3 = 6.5891 * 10.3959 = 6.5891 $(etsba)^3$.

Such a theoretical standard of Talmudic measurement units of volumes does not practically exist in the Talmud, but it seems very similar to a standard of measurement units of capacity mentioned by Rabbi Youssa in *Talmud Yerushalmi*[51] according to, which

1 *revi'it* = 2E * 2E * 1.8333 E = 7.333 $(etsba)^3$. (3)

Tossafot on B. Pesahim 109a could not imagine that this relation describes another standard of measurement units of volumes than that of Rav Hisda and they wanted to explain it using fictitious units of length of Tsipori[52] to identify it with the formula of Rav Hisda.

If we multiply equation (3) by 3840 we get the relation corresponding to (1):

1 Mikveh = 40 *se'ah* = 960 Log = 3840 *revi'it* =28159.872 $(etsba)^3$ = (28159.872 / 13824) $(cubit)^3$ = 2.037 $(cubit)^3$

Hence: 1 Mikveh = 40 *se'ah* = 2.037 $(cubit)^3$. (4)

We have thus 1 *revi'it* = 7.3333 * $(2.1825)^3$ = 7.3333 * 10.3959 = 76.2364 cm^3

1 Log = 304.9452 cm^3 and 1 Mikveh = 960 Log = 292.747 liters. The capacity of the Log is 304 cm^3 about 11% above Hemina of 274 cm^3. The

50 See Table 1 towards the end of the paper.

51 Y *Pesahim* X: 1 (near the end), Y. *Shekalim* III: 2 and Y. *Shabbat* VIII: 1.

52 See Ajdler (2009), p. 32 note 68.

standard defined by Rabbi Youssa is thus very near to the theoretical standard of the small unites of measure of volume.

● *Remark*

If we consider the following variant of formula (3):

1 *revi'it* = 2E * 2E * 1.8 E = 7.2 (*etsba*)^3. (5)

and we multiply both members by 3840, we find:

1 Mikveh = 40 Sa'ah = 960 Log = 3840 Revi'it =27648 (Etsba)^3 = (27648 / 13824) (cubit)^3 = 2 (Cubit)^3

Hence: 1 Mikveh = 40 *se'ah* = 2 (cubit)^3. (6)

Because of this result we can seriously ask ourselves if Rabbi Youssa didn't have in mind the formula (6) , instead of formula (4), when he wrote formula (3).

Furthermore, this result confirms us that Rabbi Youssa used our standard of units of length or at least a standard very near to it. Indeed, only the use of that standard of units of measure of length allows this standard to get a Log nearing to the volume of the Roman unit Hemina and the formula (6).

Strong points of this standard of the small Talmudic units of volume.

- In the present standard 1 *revi'it* = 76.2363 cm^3 and 1 egg = 50.8242 cm^3, exactly the volume of an average hen's egg.
- Some quotations of Josephus and some translations in the Septuaginta follow the small units of measure of volume.
- This standard of small units of volume allows explaining the data about the *Yam shel Shelomo* and its contents of 2000 Bat = 150 Mikveh.
- Many Rishonim adopted small units of measure of volumes. Among them we can note that Rashi,[53] Rabbeinu Gershom[54] and Rabbeinu Hananel[55] write that 1 Log = 1 Litra,[56] in contradiction with Mishna *Terumot* X: 8.

53 *Eruvin* 29a and *Hulin* 110a: ¼ Kav = 1 Log = 1 Litra.

54 *Hulin* 110a: ¼ Kav = 1 Log = 1 Litra. *Baba Batra* 89b : ¼ Kav = 1 Log = 1 Litra.

55 *Eruvin* 82b: 1 Kav = 24 Ritel = 24 Litra. Hence 1 Litra = 1 Ritel = 1 Log

56 However, the denomination Litra doesn't have the same meaning for the two first Rabbis

- Some references in the translation of the Bible by Jerome and the Septuaginta follow the smaller denomination.
- The archeological discovery[57] of a pot with a capacity of about 22 liters bearing the inscription Bat seems to be an indisputable proof that the standard of small units of capacity was not only a theoretical system but that it really existed despite the different Talmudic quotations mentioned above, which gives the impression that the Talmud favored the standard of the great units of capacity. But from the other side, it must not be exaggerated: it proves that the standard of the small units of capacity was in use in some areas in the time of the Mishna, but it does not prove that it was the only standard in use and that the other standards mentioned in the Talmudic texts had no practical existence.
- Asher Grossberg, the renowned researcher of the old miqva'ot of the Mishna period, focused my attention on the Mikveh of Massada, which had a working volume of 420 l. probably corresponding to a theoretical volume of about 332 l. or even less. This volume is much less than the theoretical volume of 40 *se'ah* = 524 l. according to the first standard (large units of volume) and even less than 431 l. according to the third standard of the average units (see further). Only the standard of the small units allows justifying this Mikveh.

Although the people of Massada were behaving according to the highest standards of purity, the volume of their Mikveh agreed with the standard

as for the third. The two first consider probably a litra of 96 * 3.50 = 336 gr. Indeed, Rashi didn't probably know the Roman denarius of 3.41 gr but he had probably at his disposal some Sela coins. R. Hananel considered probably a litra of 100 * 4.25 = 425 gr. The contradiction between the evaluation of Litra by Rashi regarding the Mishna in *Terumot* was noted in the commentary *Melekhet Shelomo* on that Mishna.

57 See "The Enigma of the Biblical *Bath* and the System of Liquid Volume Measurement during the First Temple Period", Oded Lipschits, Ido Koch, Arie Shaus, Shlomo Guil, https://www.tau.ac.il/~ashaus/2010_UF_t.pdf
See also "Vessels and Measures, The Biblical Liquid Capacity System", Raz Kletter, Israel Exploration Journal *IEJ* 64 (2014), 22-37.

of the small units of measure but it was not in agreement with our present halakhic requirements and standard.

Finally, we must note that this standard of Rabbi Youssa was disregarded by most rabbis.

It was nevertheless well known as it is described by Rabbi Eleazar ha-Kalir[58] in his piyut,[59] which is read in all Ashkenazi and Polish communities on Shabbat *Shekalim*. The only serious attempt to understand this standard of Talmudic measurement units was made by Tossafot on B. *Pesahim* 109a.[60] and in the related super commentaries. We devoted two pages, in little characters to its elucidation.[61]

Weak points of this standard of the small Talmudic units of volume.

- We have examined several Talmudic passages, which support that the Log was in fact equal to the Greek Kestes and corresponded to the larger denomination.
- The theoretical volume of the Mikveh is 291.22 l. or 2.037 $(Cubit)^3$. This volume has a section of 1 cubit * 1 cubit and a height of 2.037 cubit. If a man of 75 kg submerges, the apparent volume of water is 291.22 + 75 = 366.22 l., and the apparent height of the water is 2.55 cubit = 2.55 * 52.38 = 133.57 cm. It appears that it is impossible to submerge in a standing position. Note that although the situation looks strange, this is not a reason to disqualify this Mikveh.[62]

58 The greatest and most prolific of the early Palestinian *paytanim* living in Tiberias around the first half of the eighth century.

59 אז ראית וספרת, יוצר לפרשת שקלים.

60 בד״ה: רביעית של תורה.

61 Ajdler (2009) pp. 32-33. This interesting Tossafot teaches us much about their skill in arithmetical calculation. Nevertheless, they didn't succeed giving a satisfactory explanation. They did not understand that Rabbi Youssa described another standard of units of volumes.

62 *Shulhan Arukh Yoreh Deah* 198: 35 and 36. Indeed some Mikva'ot examined by Asher Grosberg must be used in lying position.

5. The Standard of Small Talmudic Units of Volume Appears Already in the Tossefta

The Tossefta on *Massekhet Kelim, Baba Metsia*, V: 1 is quite sibylline and the commentators are very reserved and not prolix in its elucidation.

In the first part of this Braïta, the stamist[63] mentions the minimum volume of the Mikveh: 1 cubit (length) * 1 cubit (breadth)* 3 cubits (height). This seems to be the first historical mention of this dimension and of the rule ascribed to Rav Hisda.

In the second part of the Braïta, Rabbi Yossi[64] mentions the case of the Sea of King Solomon and explains that the Sea contained 2000 Bat liquid, but it was possible to store 3000 Bat by considering the volume of solid stored in the tank of the Sea and above the building of the Sea.

The most likely explanation is the following: According to Tana Kamma, who considers that 1 Mikveh = 960 Log = 3 $(\text{cubits})^3$ =13.3333 Bat,[65] it is impossible to store 2000 Bat of liquid in the Sea. Thus, the introduction of the Sea in the debate is a manner to contradict the volume of the Mikveh of Tana Kamma. By contrast, according to Rabbi Yossi, who champions the equation 1 Mikveh = 13.333 Bat ~ 2 $(\text{cubits})^3$ this is in fact possible. I assume that the position of Rabbi Yossi was already the same as R. Youssa.[66]

Let us examine the problem in detail. The Sea of Solomon is a cylinder of external diameter of 10 cubits, internal diameter of 10 Cubits – 2 Handbreadths = 9.6667 Cubits and of height of 5 cubits.[67] The inner volume is $V = (3.1416 / 4) * (9.6667)^2 * 5 = 366.958$ $(\text{cubits})^3$.

The volume of the Mikveh of *Hakhamim* is 3 $(\text{cubits})^3$, while that of

63 The anonym author.

64 Likely Rabbi Yossi ben Halafta, one of the outstanding teachers of Rabbi Yehuda ha-Nassi. He belonged mainly to the second century.

65 1 Bat = 72 Log, 1 Mikveh = 960 Log = 13.333 Bat.

66 One standard of the small units of volume seems enough to make us happy! Rabbi Youssa is a Palestinian amora of the first half of the fourth century, probably the theoretician and the founder of the fixed Jewish calendar, ascribed to Rabbi Hillel.

67 Those are the dimensions generally accepted by the authors in their calculations.

Rabbi Yossi is probably similar to that of Rabbi Youssa[68] and is about 2 (cubits)^3.

Thus, according to *Hakhamim* the Sea holds 122.32 Mikveh = 1630 Bat and according to Rabbi Yossi and Youssa, it contains 183 Mikveh = 2446.33 Bat.

According to the former calculation we see that Rabbi Youssa had enough reserve to considering the thickness of the bottom of the tank and even to increase the thickness of the external wall. By contrast the situation is problematic according to *Hakhamim* and Rav Hisda and therefore in B. *Eruvin* 14b Rami bar Ezekiel was obliged to propose a far-fetched solution to reach the required quantity stored in the Sea.

6. The Third Standard of Talmudical Measurement Units of Volume and Capacity[69]

This is the standard generally considered in the *Talmud Bavli* and by all the Rabbis of the History, from the Talmudic times until today. The fundamental formulas binding the units of length and volume are:

1 Mikveh = 40 Sa'ah = 3 (Cubit)^3. (7)

and 1 *revi'it* = 10.8 (*etsba*)^3. (8)

The basis of this standard is much more difficult to find and explain. It is clearly a standard intermediate between the two former standards but the principle underlying it, is not clear. One point is certain, this standard is ancient and genuine, it is not the result of a mistake. Indeed, it appears already in the Tossefta[70] and it was thus existing at the time of the Mishna and Tossefta.

When we consider that the volume of the Log is about 0.546 l.[71] in the

68 Rabbi of the fourth century, probably the main craftsman of the modern Jewish calendar. He was the author of the formula (3).

69 See Table 1 towards the end of the paper.

70 *Tossefta* on *Massekhet Kelim*, *Baba Metsia*, V: 1

71 About a Sextarius or a Kestes.

first standard and about 0.305 l.[72] in the second standard, it appears that the volume of 0.449 l. for the Log according to the third standard (when we consider *etsba* = 2.1825 cm), is near to the mean value of the first two values. Maybe this was the raison dêtre of this third standard because of the hesitancy between the two first standards. If this is the explanation of this third standard, then again it would be an indirect justification of the propound standard of the Talmudic units of length that we champion. The main drawback of this standard of measurement units is that it does not carry out the rule enunciated in Mishna *Kelim* 17:11 that the units of capacity of dry material and of liquid is in correspondence with the Roman standard of units of capacity. The volume of 0.449 l. has no correspondence in the Italian standard.

7. Historical Record of the Rabbinical Standards of Measure

More than in any other area of *halakhah*, the forgetting of the preceding standards was the consequence of so many factors, debasement, political changes, emigrations,[73] modifications of the standards imposed by the authorities. The effects of this forgetting were felt in the different areas of the standards of measure.

For example, the Rabbis of Babylonia from the time of the last *Amoraïm*, the *Saborayim* and Gaonim onwards forgot the knowledge of the units of weight and coinage and assimilated the Dinar (coin and unit of weight) to the weight of an Arabic golden coin of 4.25 gr. instead of 3.41 gr. (weight according to the Roman standard of weight) or 3.50 – 3.54 gr (weight of a Dinar according to Tyrian standard) and fixed the weight of the Shekel to 17gr. instead of 14.17 gr (according to the Tyrian standard). Similarly, they forgot in Babylonia, probably very early, the significance of the Roman Mile equal to the Mile quoted in the Talmud (the Talmudic Mile) and

72 About a Hemina (11% higher).

73 The Babylonian Talmud was the main element of the tradition. Unfortunately, the Palestinian traditions, especially the transmission of the standards of measures did not resist to this emigration and was forgotten in this completely different social environment.

representing the *tehum* Shabbat, the walkable distance on Shabbat. As soon as they forgot the relation between the Mile and 2000 cubits, i.e., that the Mile is the diagonal of the square of 2000 cubits side and its major function in the Talmudic standard of the units of length, the whole standard was in jeopardy.

They were obliged to revolve toward the natural definitions of the cubit or *ama*, the handbreadth or *tefah* and the thumb or *etsba*. They even got to that point that they confused the *tehum* Shabbat with the length of 2000 cubits and the length of a step with the length of one cubit!

Similarly, in Babylonia they forgot the Roman standards of the units of capacity as well as the Talmudic units of capacity, the Log and the *revi'it* and above all the existence, and of course the details, of two out of the three standards of Talmudic measurement units of capacity, precisely the two standards of the Palestinian geographical extension, which are not even mentioned in the *Talmud Bavli*.

The only element which was not forgotten was the formulas (7) and (8) of Rav Hisda. But while the third standard of units of measure of capacity was at the origin a precise and unique standard, it became now a plain formula binding the units of length with those of capacity, in which it was possible to incapsulate an infinity of standards with units of length and capacity bound by the formula of Rav Hisda. As we show different theoretical standards of rabbinic units of length and capacity were created by the rabbis, from the Middle Ages until nowadays and recently two noted rabbis defined a standard of large units of length and capacity (Hazon Ish) as well as a standard of small units of length and capacity (Rabbi Abraham Hayyim Naé). All these standards were supposed to emulate the unique Talmudic standard of the mean units of length and capacity, which we call the standard of Rav Hisda.

Through lack of more precise data, the rabbis must fall back on the natural values of the two basic denominations, the *etsba* and the *Beitsah*, the thumb and the egg, in order to behold likely sizes for the different dimensions.

Champions of large units of capacities as well as champions of small

units of capacities were now obliged to work with the same formulas (7) and (8) and therefore the choice of large or small units of capacities, necessarily imposed large or small units of length.

Similarly, the choice of the units of weight had a direct influence on the units of capacity because the volume of the Log was expressed as the weight of its contents in water and hence on the units of length and volume. So, it appears, the whole Talmudic metric system was in disarray. Nevertheless, the rabbis succeeded in using natural values defined reasonably, defining acceptable standards, but never without some drawback. Of course, the champions of the large units of capacity and those of the small units, who had before, in the time of the Mishna, their own standard of units of capacity coexisting with the same and unique standard of units of length, had now the obligation to fulfill the same and unique formulas (7) and (8) with unpleasant consequences on the units of length. All the rabbis had their own standard of measurement units. We will examine in detail the standard of two among the most illustrious of them and then the two most recent, which are accepted in the twentieth and the beginning of the twenty first centuries.

8. Maimonides' Metric System, Emulating the Talmudic Standard II of the Small Units[74]

The data of the metric system of Maimonides, based on the principle of small units of length and capacity were included in his *Hibbur* and adopted by *Shulhan Arukh* and so entered *halakhah*. The formula (7) was introduced by Maimonides in *Hilkhot Kelim* 3: 4 and in *Hilkhot Mikva'ot* 4: 1. He ruled[75] that the *revi'it* has a capacity of 17.5 * 4.25 = 74.375 gr. of water, hence a capacity of 74.375 cm^3.

He ruled[76] also that the Litra of water weighs 35 dinars, the *revi'it* 17.5 dinar and the Log 70 dinar. Thus, the volume of the Litra is 148.75 cm^3, that

74 See Table 1 towards the end of the paper.
75 Hilkhot *Eruvin* 1: 12.
76 Hilkhot *Eruvin* 1: 12.

of the Log is 297.5 cm^3 and that of the *revi'it* is 74.375 is cm^3. We note that the *revi'it* of Maimonides is very near to the *revi'it* of the ancient standard of the Talmudical small units of measure of volume and capacity (Rabbi Youssa) of 76.2363 cm^3.[77]

In fact, the *Shulhan Arukh* ruled according to Maimonides, without nevertheless settling the dispute, as the champions of the large units of capacity did not relent, on the contrary!

In *Shulkhan Arukh, Yoreh Deah* 294:6 it writes: 1 *Ma'ah* = 16 Barleycorn= 0.25 Dirham. We know that in Maimonides' *Hibbur*: 1 Dinar = 96 barleycorn = 4.25 gr. and 1 Dirham = 64 barleycorn, hence 1 Dirham = (2/3) * 4.25 = (1/6) Sela = 2.8333 gr. In *Shulhan Arukh, Yoreh Deah* 305:1 it writes 1 Sela = 120 *Ma'ah* = 30 Dirham. Hence 1 *Sela* = 17 gr. In *Shulhan Arukh, Yoreh Deah* 324:1 it writes that the volume of flour imposing the deduction of *Hallah*, which is a volume of flour of 43.2 eggs, which is equal to 1 Issaron = 7.2 Log = 28.8 *revi'it*, weighs, in the case of Egyptian flour, 520 Dirham. This weight corresponds to 86 2/3 Sela = 1473.33 gr. Rema adds that it corresponds to a volume of about 311, 11 $(etsba)^3$. All these data were already given in Maimonides' *Hikhot Bikurim* 6: 15.[78]

From these elements we can conclude that *Shulhan Arukh* fully endorses the complete metric system of Maimonides, weight, coinage, and capacity. But as already mentioned, this did not settle the dispute. Even those who accepted Maimonides' conclusions must surmount the difficulty translating

77 But this is rather a pure chance because the two standards rest on completely different assumptions.

78 In fact, 1 *revi'it* = 10.8 $(etsba)^3$ and therefore 28.8 *revi'it* = 311.04 $(etsba)^3$. The data given by Maimonides correspond to a quantity of Egyptian flour of 28.8 *revi'it* volume weighing 520 Dirhams, hence a weight of 520/28.8 = 18.0556 Dinar/*revi'it*. Furthermore, the *revi'it* of water weighs 26.25 Dirham (*Hilkhot Eruvin* 1:12). Therefore, the density of Maimonides' flour was 18.0556/ 26.25 = 0.6878 kg/l. Today we get a density of 0.58 – 0.6 thanks to mechanical improvements. Therefore, according to Maimonides, the volume of flour requiring the deduction of *Hallah* is 74.375 * 28.8 = 2142 cm^3 and it weighs 2142 * 0.59 =1263.78 gr. rounded off to 1250 gr. The same calculation according to *Hazon Ish* gives: Volume: 43.2 * 100 = 4320 cm^3 weighing 4320* 0.59 = 2548 gr. In fact, they pronounce the benediction from 2250 gr. onwards.

Maimonides' data in local units of measure.[79] But after that the contradiction between the natural measures of thumb and eggs resulting from the fundamental formula 1 *revi'it* = 10.8 (*etsba*)^3 was revealed, the champions of the great units of measure of capacity appeared. In fact, this was a long process. In the very beginning of the 14th century Rabbi Solomon ben Menahem Meiri of Perpignan, already noted in *Beit ha-Behira*,[80] that the measure of volumes by the system based on *etsba* is safer than by the hen's eggs.

During the 15th century R. Simeon ben Tsemah Duran noted for the first time that the Mikveh determined by the volume of three cubic Cubits, measured from the natural breadth of thumb, leads to much bigger eggs than normal average-sized eggs. The problem took its full extent in Europe[81] among the Ashkenazi Rabbis, at the end of the 18th century when R. Ezekiel Landau from Prague noted that the volume of pastry, which obliges to deduct Hallah and corresponds to the volume of 43.2 eggs, is half the volume of 28.8 *revi'it* measured by the rule: 1 *revi'it* = 10.8 (*etsba*)^3. This was the starting point of the new school of the champions of the large units of capacity.

It is interesting to note that no one ever questioned the validity of the formula of Rav Hisda and realized that this formula is valid only for one unique standard based on the correct values of E, *etsba* = 2.1825 cm and *revi'it* = 112.2759 cm^3 representing a standard intermediate between the large and the small unites of volume.

a) The standard of units in the Hibbur[82]

The metric system described in his *Hibbur* represents Maimonides' final

79 Apparently, this seems the main reason of the work of R. Hayyim Naé.

80 *Eruvin* 83b and *Pesahim* 109a.

81 The problem was still raised before in different instances. See Benish (1987) pp. 63 – 68 and Weiss (1984) p. 372. See also the introduction to *Mikraot Gedolot*, Venice 1648.

82 Maimonides' great halakhic composition, also called *Mishneh Torah*, and later also, *Yad ha-Hazakah*.

position on the subject. It differs slightly from his position in his commentary on the Mishna.[83] The basic assumptions are the following: Maimonides follows the metric system of the Gaonim and the measure made by Rabbi Hilaï Gaon. He identifies the weight of the Talmudic silver Dinar with the weight of the Arabic Gold Dinar weighing 4.25 gr. He writes the weight of 1 *revi'it* water is 17.5 Dinar or 17.5 * 1.5 = 26.25 Dirham because 1 Dinar = 1.5 Dirham and 1 Dirham = 2.8333 gr. This result is equivalent to the measuring made by Rav Hilai Gaon of Sura, according to which, the weight of 1 average egg is 16.6666 Babylonian Zuz.

The Babylonian Zuz is connected to the Arabic Dinar by the following relation:

10 Babylonian Zuz = 7 Arabic Dinar. This relation is quoted in the commentary of Rashi on *Gemara Bekhorot* 50a related to the text of the Gaonic interpolation:

והנך לפדיון הבן [דהוי להו עשרין מתקלי במתקלי דינרא דאינון עשרין ותמניא זוזי ופלגא ופלגא דדנקא]

The 20 Tyrian Dinars of Pidion ha-ben are equivalent to (28 + 0.5 + 1/12) = 28.5833 Babylonian Zuz = Babylonian Dirhams. More precisely 20 Tyrian Dinars = 20 /0.7 = 28.5714 = (28 + 0.5 + 1/14) Babylonian Dirhams.

Rashi explained the relation between the Tyrian dinar and the Babylonian Dirham in Bekhorot 50a at the 24th line:

דכיוון דשבע צורים עושין עשרה בבליים...

7 Dinars of the Tyrian Standard have the same weight as 10 Byabylonian Dirhams.

Thus 1 Babylonian Zuz weighs 0.7 Dinar = 0.7 * 4.25 = 2.975 gr.

The average egg of Rav Hilai Gaon weighs 16.6666 * 2.975 = 49.5833 gr and his *revi'it* weighs 49.5833 * 1.5 = 74.375 gr = 17.5 Dinar * 4.25 gr.[84]

83 See Ajdler (2009) p. 45.

84 In all this procedure, it is assumed that the density of a hen's egg is 1. In fact, things are more complicated. A fresh egg has a density of 1.033 (The Avian Egg, A.L. Romanoff and A.J. Romanomanoff, 1949 and 1.024 (for an egg of 50 gr., The Condor 1974). This density decreases slowly, and that decrease can reach 10% in 3 weeks. Thus, after about

The weight of the egg of Maimonides is thus exactly that of Rav Hilaï Gaon[85] and his metric system appears to coincide with that of the Gaonim. From the *revi'it*, we find *etsba* = 1.9025 cm by the fundamental relation (8): 1 *revi'it* = 10.8 (*etsba*)^3.

Thus, apparently Rambam accepted the volume of an average egg, as measured by Rabbi Hilaï Gaon as departure point and calculated *etsba* through the formula of Rav Hisda and accepted probably that the value of *etsba* is a conventional unit of measure, slightly smaller than the natural value 24 mm.

We also deduce the following formulas: 1 revi'it = 17.5 Dinar. [86] (9)

1 Litra = 2 revi'it = 35 Dinar. (10)

1 Log = 4 revi'it = 70 Dinar. (11)

The formula 1 Litra = ½ Log recalls a similar formula in Mishnah *Terumot* X: 8 but the similarity stops here. In fact, Rambam called the half Log, Litra, but the three formulas are experimental and are based on the weighing of Rabbi Hilaï Gaon and of course on the weight of the Dinar adopted by Rambam and Gaonim, 20% higher than the Tyrian Dinar and 25% higher than the Roman Dinar. The formula (10) is nevertheless surprising as we are accustomed to 1 Litra = 96 dinar or 100 dinars.

In fact, Maimonides changed the definition of the Litra regarding the traditional denomination Litra in the Talmudic metric system and the denomination Libra in the Roman metric system.

As mentioned above, Rambam adopted the weight of the Dinar fixed by the Gaonim, i.e., a Dinar of 4.25 gr. And a Shekel or Sela of 17gr.

Now we know that the extant coins of Shekel weigh from 13 to 14 gr and their theoretical weight is 14.16 gr according to the Tyrian standard.

4-6 days the egg floats because its density reaches 1. The phenomenon depends much on the conditions of conservation.

85 References to the weighing of Rav Hilaï Gaon can be found in Benish: *Midot ve shiurei Torah*, Benei Berak 1987, pp. 216 – 217. Four Gaonim named Hilaï were in function in Sura between 694 C.E. and 898 C.E. I did not find more precise elements about the one responsible for this measure.

86 From the formula of Rav Hisda we find *etsba* = 1.9025 cm and C = 45.6610 cm.

Furthermore, we know that Rambam knew these coins and had probably some exemplars at his disposal as we learn from a quotation from one of his responsa.[87] The explanation seems to be the following: Rambam considered that the coins at his disposal were the ancient Jewish Shekalim from before the re-evaluation of 20% discussed in *Bekhorot* 50a. Therefore, the Talmudic Shekel must be fixed at 14.16 * 1.2 = 17 gr.

b) The standard of units in the Commentary on the Mishna

In his commentary on the Mishna, Rambam presents a slightly different metric system. From his commentary on Mishna *Bekhorot* 8: 8, we learn that 1 Dirham = 61 Barleycorn. From his commentary on *Eduyot* 1: 2 we learn that 1 *revi'it* water weighs 27 Dirham. We have then in the commentary: 1 Dirham = (61/96) * 4.25 = 2.70 gr and 1 *revi'it* water = 27 * 2.7 = 72.9 gr instead of 17.5 * 4.25 = 74.375 gr.[88] Hence an increase of 2%.

The change in Maimonides' position about the weight of the *revi'it* of water can be understood. Despite his own measure, as he underscored, Rambam preferred to accept, in his *Hibbur,* the value of Rabbi Hilaï Gaon and the Gaonim. But the other changes raise great interrogations and difficulties because we have no knowledge of a major change of the Dirham at this epoch. Furthermore, a re-evaluation of the Dirham, would be something exceptional. Such a change in the value of the Dirham during Maimonides' lifetime should have been underscored by him. Furthermore, we don't understand why the weight of one Omer of Egyptian flour remained unchanged at 520 Dirham while the Dirham was re-evaluated by 4.92%?[89]

The relation today universally accepted and ruled in *Shulhan Arukh*,[90] that 1 Dinar = 1.5 Dirham is derived from Hilkhot *Bikkurim* 6: 15 where he

87 Responsum n° 468 in *Tshuvot ha-Rambam*, Joshua Blau, Jerusalem 1960, vol 2, p, 513.

88 Exactly in agreement with the weighing of Rabbi Hilai Gaon of 49.5833 gr

89 These problems were already raised in Jacob Weiss*: Midot U Mishkalot shel Tora,* Jerusalem 1984, p. 159.

90 *Shulhan Arukh Yoreh Deah* 294, 6 and 305, 1 and 324:1. See also *Kessef Mishneh* on Hilkhot *Bikkurim* 6:15 and on Hilkhot *Kelei ha-Mikdash* 3: 3.

writes that 520 Dirham = 86 2/3 Sela = 346. 66667 Dinar.[91] I have proposed to solve this conundrum by invoking a lapsus calami in the figure 86 2/3 and correcting it to 82 2/3.[92] But I agree that this correction is not sustained by any manuscript evidence, and it would then be the author's mistake! But it would solve the contradiction of the unlikely re-evaluation of the Dirham.

Weak points of this standard:

a. The value of his breadth of thumb, of his cubit and of his *tehum* Shabbat are very small.
b. There is no connection between these values and the Talmudic Mile, which we know today, is equal to the Roman Mile.
c. The Shabbat walker covers 2000 cubits in 2000 steps. It is practically

91 If 1 Dirham = 64 Barleycorn = 2/3 Dinar, then 520 Dirham = 520 *2/3 = 346.6667 Dinar = 86.6667 Sela. If 1 Dirham = 61 Barleycorn, then 520 Dirham = 520 * 61/96 = 330.4167 Dinar = 82.6042 Sela or about 82 2/3 Sela. Therefore, as it seems difficult to explain that the weight of the Dirham changed during the end of the commentary of the Mishna and the redaction of the *Hibbur*, it is perfectly likely that the Dirham remained unchanged and that the weight of one *omer* of Egyptian flower mentioned in Hilkhot *Bikkurim* is 82 2/3 Sela instead of 86 2/3 Sela. Otherwise, the weight of the *omer* Egyptian flower should have become (61/64) * 520 = 495.625 Dirham in the *Hibbur*! Or with more precision (17.5/17.1563) * 495.625 = 505.5 Dirham, considering the undisputable increase of the volume of the *revi'it* and the *omer* by (17.5/17.1563 = 1.0202) in the *Hibbur* regarding the commentary on the Mishna. This seems to be an additional serious argument against the re-evaluation of the Egyptian Dirham during the time of the redaction of the *Hibbur*. Unfortunately, in the meantime, the *Mehaber*, Rabbi Joseph Karo already ruled in *Yoreh Deah* 294:6 that 1 *ma'ah* = 16 barleycorn = 0.25 Dirham and in *Yoreh Deah* 305:1 and 324:1 that 5 Sela = 120 *ma'ah* = 30 Dirham and therefore 1 Sela = 1.5 Dirham. Nevertheless, we must note that even, with the unchanged Dirham, the weight of the *omer* of the Egyptian flower should have been anyhow adapted to the increase of the volume of the *revi'it* to (17.5 / 17.1563) * 520 = 530.4 Dirham. In other words, whether we accept the re-evaluation of the Dirham or not, there is a problem with the weight of the *omer* of Egyptian flower in his *Hibbur,* regarding its weight in his commentary of the Mishna. Apparently the most likely assumption is that the Egyptian Dirham did not change, that we should read 82 2/3 instead of 86 2/3, that the *revi'it* was adapted to coincide with the Gaonic *revi'it* but that he considered that the consequences of this adaptation are insignificant and therefore he neglected them.

92 See preceding note.

impossible to walk with such short steps of 45.66 cm. Such a bearing belongs to an ill and weak man, who hardly drags himself along. It is almost hard to keep your balance in such a bearing.

d. His assumption, following the Gaonim, that the Talmudic Shekel weighs 17 gr. is problematic.

Strong points of this standard:

a. The capacity of his Log is 297.50 cm^3. It is close to the capacity of Hemina: 274 cm^3
b. The volume of his egg at 49.58 cm^3 is very likely.

9. Rashi's Metric System, Emulating the Talmudic Standard II of the Small Units[93]

Rashi and Rabbeinu Gershom wrote: 1 Log = 1 Litra. The weight of the Sela, at Rashi and Rabbeinu Gershom's disposal was about 14 gr.[94] or slightly less according to the condition of the coins at his disposal.[95]

Rashi and Rabbenu Gershom made a difference between Mana and Litra. This was also the case in the German coinage, which was emulating the Roman coinage. We can find a proof of this statement in the following quotations from Rabbenu Gershom in *Bekhorot* 49b and in *Hulin* 84a:

93 See Table 1 towards the end of the paper.

94 In his commentary on Ex. 21:32 and 25:39 he estimates the weight of the Shekel at half an ounce of Köln or 14.615 gr. It was probably an order of magnitude expressed in common currency rather than a precise measurement. But in fact, the weight of the Shekel at the disposal of Rashi or of Rabbeinu Gershom was probably slightly less than 14 gr. Indeed, most of the coins available were Roman coins re-struck during the revolt. Indeed, in his commentary to B. *Bekhorot* 49b and *Baba Kamma* 36b he writes that 1 Sela = 10 *esterling* corresponding to about 14 gr. This was a more precise estimation than in his commentary on *Shemot*.

95 About the metrology of Rashi, see Ajdler (2010), Talmudic Metrology V: Halakhic Coinage in the Post-Talmudic Period. See also: Laurence J. Rabinovich, Rashi's Metrology: Money, Coins and Currency from Cologne, Constantinople and the Classical Past, The Manchester 2008 Conference Volume.

דינר זהב הוי ה׳ פשוט למשקל הברזל, דינר כסף ב׳ ומחצה למשקל הברזל. בכורות מ״ט ע״ב.

והמנה הוא ליטרא וחצי אונקיה. חולין פ״ד ע״א.

The Pashut refers to the Esterling coin of 1.4 gr. Thus, according to the first quotation

1 Dinar = 2.5 Esterrling coins = 2.5 * 1.4 = 3.5 gr.

1 Ounce = 20 Esterling coins

1 Mana = 250 Esterling coins = 250 * 1.4 = 350 gr.

1 Litra = 240 Esterling coins = 240 * 1.4 = 336 gr.

Furthermore, Rabbenu Gersom wrote in Hulin:

1 Mana = 240 Esterling coins + 10 Esterling coins = 250 * 1.4 = 350 gr.

Thus 1 Log = 1 Litra = 96 * 3.5 = 336 gr or 1 Log = 0.336 l.

Reconstitution of the three original Talmudic standards of units of volume.

	Standard I: Large units of measure Mishna *Terumot*	Standard II: Small units of measure Rabbi Youssa	Standard III: Mean Units of measure Rav Hisda
Beitsah	90.96 ml	50.824 ml	74.85 ml
Revi'it	136.44 ml	76.236 ml	112.28 ml
Litra	272.88 ml	152.473 ml	224.55 ml
Log	545.76 ml	304.946 ml	449.10 ml
Kav	2.183 l	1.220 l	1.796 l
Hin	6.549 l	3.659 l	5.389 l
Sa'ah	13.098 l	7.812 l	10.778 l
Bat	39.294 l	21.956 l	32.335 l
Omer	3.929 l	2.195 l	3.233 l
Kor = Homer	392.94 l	219.170 l	322.86 l

Table 1: Reconstitution of the three Talmudic standards of units of measure of capacity. The assumptions of the different standards are the following: Standard 1, Large units of capacity: Dinar = 3.411 gr *etsba* = 2.1825 cm, cubit = 52.38 cm, Log = 545.76 ml. Standard 2, small units of capacity: Dinar = 3.411 gr *etsba* = 2.1825 cm, cubit = 52.38 cm, Log = 304.95 ml. Standard 3, mean units of capacity: Dinar = 3.411 gr *etsba* = 2.1825 cm, cubit = 52.38 cm, Log = 449.10 ml.

The four modern (post-Talmudic) halakhic standards of units of volume.

	Rambam: Small units of measure.	Rashi, assumed Small units of measure	R. Naé: Small units of measure	Hazon Ish: Large units of measure
Beitsah	49.58 ml	56 ml	57.60 ml	100 ml
Revi'it	74.37 ml	84 ml	86.40 ml	150 ml
Litra	148.75 ml	168 ml	172.80 ml	300 ml
Log	297.5 ml	336 ml	345.60 ml	600 ml
Kav	1.190 l	1.344 l	1.382 l	2.40 l
Hin	3.570 l	4.032 l	4.147 l	7.20 l
Sa'ah	7.140 l	8.064 l	8.294 l	14.40 l
Bat	21.420 l	24.192 l	24.860 l	43.2 l
Omer	2.142 l	2.419 l	2.488 l	4.32 l
Kor = Homer	214.20 l	241.92 l	248.84 l	432 l

Table 2: The three present standards of units of measure of capacity. The assumptions of the different standards are the following: 1 Revi'it = 10.8 (*etsba*)[3]. Standard 1, Rambam, small units of capacity: Dinar = 4.25 gr, *etsba* = 1.9025 cm, Cubit = 45.66 cm. Standard 2. Rashi, small units of capacity: Dinar = 3.50 gr., *etsba* = 1.9813 cm, Cubit = 47.55 cm. Standard 3, R. Naé small units of capacity: *etsba* = 2.0 cm, Cubit = 48 cm. Standard 4, Hazon Ish, large units of capacity: *etsba* = 2.40 cm, Cubit = 57.6 cm.

The formula 1 Log = 1 Litra seems also to be an experimental formula based on the estimation of the volume of average eggs.

We do not have first-hand information about the units of length of Rashi, but we can deduce them from the formula of Rav Hisda. We calculate *etsba* = 1.98 cm and *tefah* = 47.55 cm,[96] but his Log of 0.336 l. moves away from Hemina. The weak points of this standard are about the same as those of the standard of Rambam.

96 These values are close to those of Maimonides.

10. Rabbi Hayyim Naé's Metric System, Emulating Maimonides' Metric System[97]

When Rabbi Hayyim Naé proposed his metric system emulating Maimonides' metric system and the Talmudic standard of small measurement units, the correct quantification of Maimonides' data was not yet completely elucidated. Otherwise, it is difficult to understand why he began such a difficult exercise. On scientific level, the method of R. Naé is very problematic. His system is based on contradictory and anachronistic assumptions.

He departs from the relation in Maimonides' commentary on Mishna[98]: 1 *revi'it* water weights 27 Dirham. Rabbi Hayyim Naé adopts for the Dirham a weight of of 3.2 gr. based on the Turkish Dirham of the nineteenth century and the beginning of the 20th century[99] Furthermore he deducts the weight of the Dinar from the relation:[100] 1 Dinar = (96 / 64) * 3.2 = 4.8 gr. By so doing he commits not only a dangerous anachronism (confusing Maimonides' Dirham[101] with a modern Turkish Dirham of 3.2 gr) but he forgot that in the commentary of the Mishna the Dirham weighs 61 Barleycorns while in the *Hibbur* and the *Shulhan Arukh* it weighs 64 Barleycorns. He made thus an unlucky mixture: 27 Dirham for a *revi'it* water is a data in the commentary of the Mishna (instead of 17.5 * (96/64) =26.25 in the *Hibbur*) and 64 is the weight of the Dirham in the *Hibbur* (while it is 61 Barleycorns in

97 See Table 1 towards the end of the paper.

98 Commentary on the following Mishnayot: *Peah* 8: 5, Shevi'it 1: 2, *Hallah* 2: 6, *Terumot* 10: 8, *Eduyot* 1: 2, *Menahot* introduction, 5th part, *Menahot* 9: 2, *Bekhorot* 8: 8, Kelim 2: 8 and *Mikvaot* 3: 1.

99 This weight of 3.2 gr is in good correspondence with the indication given by Rabbi Hayyim Palaggi for the Dirham of his time in his *Aggada shel Pessah* with the Commentary *Hayyim Lerosh*. See Rabbi Jacob Gershon Weiss, *Midot U Mishkalot shel Torah*, Jerusalem 1984.

100 Taken in the *Hibbur* and apparently in contradiction with the commentary of the Mishna.

101 The Dirham of Rambam weighs (61/384) * 17 =2.7 gr in his commentary of the Mishna and (64/384) *17 = 2.8333 gr. According to his *Hibbur.*

the commentary of the Mishna). The result of this mixture cannot be very significative or reliable.

Thus, the main elements of his standard of halakhic measures

1 Dinar = (96/64) 3.2 = 4.8 gr and 5 Shekel = 20 Dinar = 96 gr.[102]
1 *rev'it* = 27 * 3.2 = 86.4 gr of water, hence a volume of 86.4 cm^3.
and the relation 1 *revi'it* = 10.8 (*etsba*)^3 gives us 1 *etsba* = 2 cm.

Now that Maimonides' and Rashi's metric systems are well known, we can be surprised that the metric system of Rabbi Hayyim Naé, which is not a model of coherence, remains revered and popular. In fact, we note that it is similar to the assumed and more coherent position of Rashi. Nevertheless, the weight of the Dinar of Rabbi Naé of 4.8 gr. is unrealistic overvalued.

11. Rabbi Abraham Karelitz's (Hazon Ish) Metric System, Emulating the Talmudic Standard of the Big Units of Measure[103]

Hazon Ish based himself on the reasoning of Rabbi Ezekiel Landau and constructed all his system on the average breadth of thumb of 2.4 cm and the cubit of 24 * 2.4 = 57.6 cm. This gives us a *revi'it* of 149.30 cm^3, which he rounded off to 150 cm^3. He kept the dinar of Rabbi Hayyim Naé.

Weak points of this standard.

a. The length of the breadth of thumb and of the cubit are very high and taller that the values ever proposed in history.
b. There is no connection between the length of the cubit and the Talmudic Mile, which, we know today, was equal to the Roman mile.
c. A breadth of thumb of 2.4 cm leads to a cubit of 57.6 cm! Certainly not the elbow of a man!

102 These figures are even taller than those of Rambam and Gaonim.
103 See Table 1 below.

d. The volume of his hen's egg is 100 cm^3 is huge and unrealistic. The thesis of *Hazon Ish,* according to which, the volume of hen's eggs was divided by two throughout history is undefendable. We tried to justify it by considering the volume of the parallelepiped circumscribed to this egg according to the principle of the box of artichokes in Erubin 83a. However, measuring the volume of eggs differently than in the two other standards, seems farfetched.
e. The Log exceeds the Sextarius by 10% and seems very large.
f. Drinking a *revi'it* wine (150 cm^3) or even only its majority and eating a *keza'it* matsa (33 or 50 cm^3 according to the more stringent opinion, which is generally followed), becomes very difficult for average people.
g. The weight of his Shekel of 19.2 gr. is huge and unrealistic.

The standard of *Hazon Ish* is a standard giving to the faithful the guarantee to fulfil his obligation whatever the prevalent opinion may be, rather than a scientific standard emulating the Talmudic standard of measurement units. It is difficult to understand how this standard imposed itself in the Jewish society.[104]

104 To the point that the ancient prescriptions for the deduction of *Hallah*, in the popular Siddur *Sefat Emet* (Heidenheim, edited in Rödelheim from 1806 onwards and recently in Basel) the siddur of the German *minhag* par excellence, were adapted in the last editions published in Basel to the standard of *Hazon Ish*. Therefore, the size of the dough or the pastry liable to *Hallah* has been increased and almost doubled. A new zone has been created, for which, deduction must be performed without benediction. Many domestic pastry-makings are in this intermediate quantity zone and women belonging to the Ashkenazi *minhag*, who will with the time, abide by this new rule, will completely forget the benediction of *Hallah* deduction. This situation constitutes a regrettable abandonment of an ancestral use, dating back to the dawn of time and unfortunately this case is not unique. Nevertheless, here this abandonment was freely accepted by the editor of this Ashkenazi Siddur par excellence. This is a deplorable decision, which, in keeping with the general pattern of the running toward the harshness and strictness of a concurrent tradition, is performed at the cost of the neglect of the Ashkenazi millennial traditions.

12. Summary

1. There was only one standard of Talmudic units of length. The Talmudic Mile was identical to the Roman Mile. It was the diagonal of a square of 2000 cubits side. It became the basis of the Talmudic standard of the units of length. This system is quasi-identical with the rabbinical standard of the units of length of the Polish and Russian Rabbis of the nineteenth and twentieth centuries, before the war.[105]

   1 Mile = 1481.5 m, *etsba* = 2.1825 cm, 1 cubit = 52.38 cm.

2. Three different standards of Talmudic units of capacity coexisted at the period of the Mishna and the Talmud with one unique standard of Talmudic measurement units. They correspond to three different relations between the units of volume and the units of length.

- **Standard of large units of capacity described in Mishna** *Terumot.*
  Starting point: 1 Log = 160 Dinar = 160 * 3.411 = 545.76 cm^3 = 1 Sextarius.

  Main characteristics:

  etsba = 2.1825 cm, *revi'it* = 136.44 cm^3, Log = 545.76 cm^3.
  1 Mikveh = 3.6457 $(cubit)^3$. 1 *revi'it* = 13.1244 $(etsba)^3$.

- **Standard of small units of capacity described by Rabbi Youssa.**
  Starting point: 1 *revi'it* = 2E * 2E * 1.8333 E = 7.333 $(etsba)^3$.

  Main characteristics:

  etsba = 2.1825 cm, *revi'it* = 76.24 cm^3, Log = 304.95 cm^3 ~ 1.1 Hemina.
  1 Mikveh = 2.037 $(cubit)^3$. 1 *revi'it* = 7.3333 $(etsba)^3$.

105 They identified the Techum shabbat of 2000 cubits to 1 Verste = 1067 meter, hence 1 cubit = 53 cm. This concordance is limited to the length of the cubit, but the Mile is 2000 cubits instead of 2828 cubits.

- **Standard of medium-sized units of capacity described by Rav Hisda.**
  Starting point: 1 *revi'it* = 10.8 $(etsba)^3$.

  Main characteristics:

  etsba = 2.1825 cm, *revi'it* = 112.25 cm^3, Log = 449 cm^3 ~ 1.6 Hemina.
  1 Mikveh = 3 $(cubit)^3$. 1 *revi'it* = 10.8 $(etsba)^3$.

3. During the period of the Gaonim the precedent standards were forgotten and only the two equivalent formulas: Mikveh = 3 $(cubit)^3$ and 1 *revi'it* = 10.8 $(etsba)^3$ subsisted and were remembered. They were not more considered as defining one unique standard, but they were considered as a general rule, authoritative for any metric system. The respect of these formulas complicated the finalization of a rabbinic halakhic metric system based on natural values of *etsba* and *Beitsah* and was the cause of the anomaly observed at the end of the 18th century in their metric system: when the *Beitsah* had a normal volume of an average hen's egg, then the etsba was too small and when the *etsba* had a normal value, the volume of the *Beitsah* was much too big. All the rabbinic standards of measurement units were based on small or average units until the eighteenth century. Nevertheless, here and there some reservations were made, and some rabbis proposed to increase the size of the *Miqveh,* to consider, the natural value of the breadth of thumb. The tendency increases at the end of the eighteenth century, and it experienced a lashing acceleration in the twentieth century with the development of the standard of *Hazon Ish* units.
4. Maimonides' metric system was certainly like the standard of the Gaonic units, and it remained authoritative in the Sephardic world until today. It is very similar the Standard II of the small units of measure of Rabbi Youssa except for the value of the unit of length *etsba* and the value of the Talmudic Sela.

Starting point: The weighing of Rabbi Hilaï Gaon: 1 egg = 49.58 gr. and 1 Dinar = 4.25 gr and 1 Sela = 17 gr.

Main characteristics:

etsba = 1.9025 cm, *revi'it* = 74.375 cm^3, Log = 297.5 cm^3.

5. In Germany the Rabbis of the tenth and eleventh century in General and particularly Rashi and Rabbenu Gershom had a very good knowledge of the Roman and Talmudic metric systems. This German metric system was authoritative in Europe until the end of the 18^{th} century at the outbreak of the crisis of the rabbinic standard of units of measure of length and capacity.

   Starting point: 1 Dinar = about 3.5gr. 1 Log = 1 Litra = 1 Libra = 96*3.5 = 336 gr.

   Main characteristics:

   etsba = 1.9813 cm. *revi'it* = 84 cm^3, Log = 336 cm^3

6. Today two standards coexist: the standard of the small units of Rabbi Naé which is similar to the German standard of small units of length and capacity ascribed to Rashi in the present paper and the standard of *Hazon Ish* which is similar to the ancient Talmudic standard of the large units of capacity of the Mishna *Terumot* but it overestimates the units of length and those of capacity. The standard of *Hazon Ish* superseded the more moderate and average standard in use in Poland and Russia, which rather, emulated the Talmudical standard of the average measurement units.

   a. Rabbi Naé.

   Starting point: 1 Dirham = 3.2 gr.

Main characteristics:
1 Dinar = 4.8 gr. 1 *revi'it* = 27 * 3.2 = 86.4 gr.
1 *etsba* = 2 cm. 1 Log = 345.6 cm^3

b. Hazon Ish.

Starting point: 1 Dinar = 4.8 gr. 1 breadth of thumb = 2.4 cm.

Main characteristics:
etsba = 2.4 cm. 1 *revi'it* = 150 cm^3. 1 Log = 600 cm^3

13. Conclusion

The main achievements of this paper are the original and even revolutionary proposals (more than assumptions but still not completely indisputable proofs) developed in points 1, 2 and 3 of the Summary above. Points 4, 5 and 6 are the synthesis of classical, but strongly scattered, elements. Nevertheless, our assumption that the weight of the Dirham of 61 barleycorn in Maimonides' *Commentary on the Mishna* was still a Dirham of 61 barleycorn in his *Hibbur,* and did not change at all, is original but it remains a conjecture aiming at the solution of a real conundrum. It implies, however, that when Rambam increased in his *Hibbur* his units of capacity by 2% to bring his standard of units of volume in concordance with the standard of Gaonim and the weighing of Rabbi Hilaï Gaon, he did not change the other data and considered that this change of 2% was negligible.